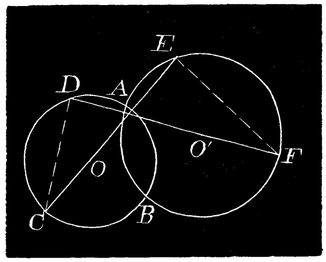
Connect by straight lines the points $D$ and $C$, and $E$ and $F$. Then the triangles $CDA$ and $AED$ are right triangles, being inscribed in a semi-circle. The two triangles are also similar, having the acute angle $EAF$ of the one equal to the acute angle $DAC$ of the other.




$\therefore$ $CA : AF :: AD : AE$, the homologous sides of similar triangles being proportional.

$\therefore$ $CA\times AE = AF\times AD$. Q. E. D.

*Remark.*—When the angle $CAF$ is a right angle $AE$ and $AD$ each equal zero. When the angle $CAF$ is less than a right angle, the point $E$ will fall on the semi-circumference $ABF$ and the point $D$ will fall on the diameter $AF$ and $CA\times AE = AF\times AD$ as before.

Also solved by *J. F. W. SCHEFFER, JOSIAH H. DRUMMOND, ROBERT J. ALEY, G. B. M. ZERR, J. A. CALDERHEAD, P. S. BERG.* and *P. H. PHILBRICK.*

10. **Proposed by ERIC DOOLITTLE, Instructor in Mathematics, State University of Iowa, Iowa City.**

If $MN$ be any plane, and $A$ and $B$ any points without the plane, to find a point $P$, in the plane, such that $AP+PB$ shall be a minimum.

**Solution by THADDEUS MERRIMAN, South Bethlehem, Pennsylvania.**

First, suppose the points to be on opposite sides of the plane; the point where the straight line joining the given points pierces the plane is the required point $P$; since a straight line is the shortest distance between two points.

Second, suppose the points to be on the same side of the plane; let $AB$ be the straight line joining them, $CD$ the projection of $AB$ on the plane $MN$, and $AC$ the perpendicular which projects $A$ on the plane. Now, produce $AC$ to $E$ making $AC=CE$, and join $E$ and $B$; the point $P$ where $EB$ cuts $CD$ is the required point. For, join $A$ and $P$, and let $Q$ be any other point in the plane; join $A$ and $Q$, and $B$ and $Q$, also $Q$ and $E$. Now, since $AC=CE$ by construction, $AP=PE$ and $AQ=QE$; consequently $BQ+QE=BQ+QA$, and therefore, since $BE<BQ+QE$, we have $AP+PB<AQ+QB$, or $AP+PB$ is a minimum.

[This demonstration is by Thaddeus Merriman, the 17 year old son of Professor Mansfield Merriman.—ED.]

Also solved by *J. H. BEACH, G. B. M. ZERR, LEONARD E. DICKSON, F. A. SWANGER, H. C. WHITAKER, P. H. PHILBRICK, A. H. BELL* and *J. F. W. SCHEFFER.*

## PROBLEMS.

31. **Proposed by Professor G. I. Hopkins, Manchester, New Hampshire.**

A field is bounded as follows: N. 14° W. 15.2 chains; N. 70°30′ E. 20.43 chains; S. 6° E. 22.79 chains; N. 86° 30′ W. 18 chains. A spring within it bears from the second corner S. 75° E. 7.9 chains. It is required to cut off 10 acres from the west side of the field by a straight fence through the spring. How far will it be from the first corner to the point at which the division fence meets the fourth side?